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(1). $c-a : b :: x : a. \quad \therefore x=\frac{ac-a^2}{b}$ ....1.

(2). $c-a : b :: b-x : c. \quad \therefore x=\frac{b^2-c^2+ac}{b}$ ....2.

(3). $x : a :: b-x : c. \quad \therefore x=\frac{ab}{a+c}$ ....3.

From the three equations, it is evident that we may obtain three proofs by this method.

IV. Let ABC be a △ right-angled at C. Extend AB to D making $BD=BC$. Draw a line perpendicular to AD at D, meeting AC produced as at E. Then will $CE=DE$, and △ AED will be similar to △ ABC.

Letting $AC=b$, $BC=a$, $AB=c$, $AD=c+a$, $DE=x$, $AE=x+b$, and proceeding as in the last case, we obtain three more proofs, making in all, thus far, 96 proofs.

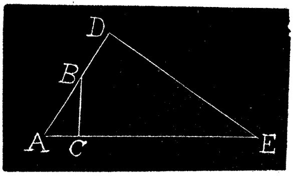

Fig. 4.

In our next paper, we shall give a method whose results reach into the thousands.

[To be Continued.]

NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

By GEORGE BRUCE HALSTED, A. M., (Princeton); Ph. D., (Johns Hopkins); Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.

[Continued from February Number.]

PROPOSITION XXIII. *If any two straights AX, BX (Fig. 27.) exist in the same plane, either they have (even in the hypothesis of acute angle) a common perpendicular, or prolonged toward either the same part, unless sometime at a finite distance one strikes upon the other, they mutually approach ever more toward each other.*

Proof. From any point A of AX is let fall to the straight BX the perpendicular AB. If BA makes with AX a right angle, we have the asserted case of the common perpendicular. But otherwise this straight makes toward one or the other part, as suppose toward the parts of the point X, an acute angle.

So in the aforesaid straight *AX* between the points *A* and *X* any points *D*, *H*, *L* are designated, from which are let fall to the straight *BX* the perpendiculars *DK*, *HK*, *LK*. If any one angle at the points *D*, *H*, *L* be acute toward the parts of the point *A*, it follows (from the preceding) that *AX*, *BX* will have a common perpendicular.

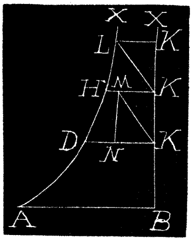

Fig. 27.

But if every angle of this sort be greater than acute; either some one will be right, and thus again we will have the asserted case of the common perpendicular, since all angles at the points *K* are supposed right; or all those angles toward the parts of the point *A* are obtuse, and therefore all therewith acute toward the parts of the point *X*, and so again I argue: Since in the quadrilateral *KDHK* the angles at the points *K* are right, but the angle at the point *D* is acute, the side *DK* will be (from Cor. II. after P. III.) greater than the side *HK*.

In a similar way the side *HK* is shown to be greater than the side *LK*; and so always, comparing to each other perpendiculars from any ever higher points of *AX* let fall upon the other *BX*. Wherefore *AX*, *BX* mutually approach each other ever more toward the parts of the point *X*: Which is the second part of the disjunct proposition.

From all which follows that any two straights *AX*, *BX*, which exist in the same plane, either have (even in the hypothesis of acute angle) a common perpendicular, or produced toward either the same part, unless sometime at a finite distance one strikes upon the other, mutually approach each other ever more.

Quod erat etc.

Corollary I. Hence the angles toward the base *AB* will be always obtuse at each point of *AX*, from which is let fall a perpendicular to the straight *BX*: will be, I say, always obtuse, as often as those two *AX* and *BX* mutually approach each other ever more toward the parts of the points *X*; which indeed ought to be understood in a sane way, of course, of perpendiculars let fall before the aforesaid meeting, if perchance one should strike upon the other at a finite distance.

Scholion. I see indeed that it may be here inquired, in what way can be shown the existence of that common perpendicular, as often as any straight *PFHD* (Fig. 28.) meeting two *AX*, *BX* in points *F*, and *H*. makes toward the same parts two internal angles *AHF*, *BFH*, not themselves indeed right, but nevertheless together equal to two rights. But behold that common perpendicular geometrically demonstrated.

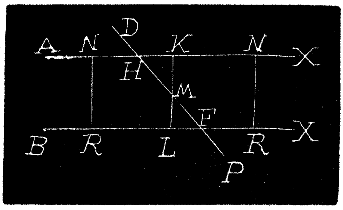

Fig. 28.

FH being. bisected in *M*, perpendiculars *MK*, *ML* are let fall to *AX* and *BX*. The angle *MFL* will be equal (Eu. I. 13) to the angls *MHK*, which indeed is supposed to make up two right angles with the angle *BFH*. Moreover the angles at the points *K* and *L* are right ; and again MF, MH are equal. Therefore (Eu. I. 26) so are the angles FML, HMK equal. Wherefore the angle HMK makes two right angles with the angle HML, since with this the angle FML (Eu. I. 13) makes two right angles. Therefore (Eu. I. 14) KML will be one continuous straight line, consequently the common perpendicular of the aforesaid straights AX, BX. Quod erat etc.

[To be Continued.]

INTRODUCTION TO SUBSTITUTION GROUPS.

By G. A. MILLER, Ph. D., Leipzig, Germany.

[Continued from February Number.]

The Construction of the Primitive Groups.

We have shown that all the intransitive and the non-primitive groups of a given degree, may be made to depend upon groups of a lower degree. We shall soon prove a similar property of the primitive groups.

It must however not be inferred that this will solve, in a satisfactory manner, the problem of constructing all the groups of a given degree. The elementary methods to which we have confined ourselves require a large number of trials if the degree is large. Some briefer methods will be given later but even these will only tend to make the construction of all the groups of a given degree practical for somewhat larger degrees.

It is not difficult to give general theorems which include all the groups of a given type, as, for instance, the theorem at the end of our discussion of the construction of the non-primitive groups ; but new types arise continually and no non-tentative method by means of which all the groups of any given degree may be found has yet been published.

We proceed to prove some theorems which apply to all transitive groups but are especially useful in the construction of primitive groups. Unless the contrary is stated the symbols G, g, and n will respresent respectively the group under consideration, its order and its degree.

Let us consider the transitive group G which contains the letters $a_1, a_2, \ldots\ldots, a_n$. The substitutions of G which do not contain a_1 (*i. e.*, those which replace a_1 by itself) may be represented by

$$s_1,\ s_2,\ \ldots\ldots,\ s_r \equiv G_1.$$

As every group must contain the identical substitution if the number of its